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Petri net extension for traffic road modelling

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Abstract—**Traffic flow modelling is an essential step for designing and controlling the transportation systems. It is not only necessary for improving safety and transportation efficiency, but also it can yield economic and environmental benefits. Consider the discrete and continuous aspects of traffic flow dynamics, hybrid Petri nets have proved to be a powerful tool for approaching this dynamics and describe the vehicle behavior accurately since they include both aspects. A new extension of hybrid petri net is presented in this paper for generalizing the traffic flow modelling through taking into account state dependencies on external rules which can be timed and also nondeterministic time such as stop sign or priority roads. Moreover, a segmentation of roads is proposed to deal with the accurate localization of events.**

Index Terms—**Traffic flow models, Road network, Petri nets, Batch Petri nets.**

I. Introduction

The number of vehicles is a growing phenomenon which causes congestion, pollution (air and noise), traffic accidents and economic loss through long trip times, and so on [1], [2]. Studies and reports have been published consecutively to reveal the economic and environmental impact of traffic problems [2], [3], [4], [5]. This explains the increasing attention that Intelligent Transport Systems (ITSs) has attracted during the last decades. ITSs are a key element in both enhancing human life and improving modern economy; they majorly aim to optimize the road traffic flow [3], [6], provide route guidance, manage the road network capacity, improve safety of drivers, reduce energy consumption and enhance environmental quality and others.

Modelling traffic is a key component of ITSs for understanding the dynamic changing of the traffic flow and hence optimizing the performance of travels [7]. Modelling the traffic in the most reliable way, provides a framework to better investigate real-time road state and predict accurately the future traffic [8], [9], [10].

In general, a desirable model must comply with the following requirements:

- It must be in accordance with the traffic flow;
- It must be flexible, to take into account sufficient parameters which can characterize traffic flow and can be interpreted in traffic terms;
- It must be simple, represent accurately different situations and behaviors and deal with random dynamics of traffic flow in road networks.

Traffic flow, is a large scale and complex dynamical system that possesses continuous states (traffic behavior at highway or single stretch of road) and discrete states (traffic behavior at intersections). The dynamic evolution of traffic have been approached in literature using stochastic models [11], [12], [13], mathematical models derived from partial differential equations and then later by hydrodynamic theory of fluids flow [14], [15], [16], [17]. However, traffic phenomenon can be considered as a hybrid system, where a discrete event system (DES) and a continuous time system interact. Petri nets (PNs) are primarily designed for discrete modelling especially to deal with issues such as parallelism, nondeterminism, concurrency and synchronization, etc. and then are extended with continuous dynamics [18]. As a consequence combining the advantages of both classes leads that PN can be suitably derived for traffic modelling.

The rest of the paper is organized as follow. In Section 2 we present the necessary backgrounds information to understand the rest of this paper. Related works are highlighted in Section 3. Whereas, Section 4 provides an improved Petri net model based on batch concept to take into account special road traffic cases and deal with dependencies on external events.

II. Background on triangular batches Petri net

We introduce in this section the theoretical background of our modified batches Petri nets version

A. Petri Nets

A Petri net [19] is a graphical model that consists of three kinds of objects: places, transitions and directed arcs, and provides both dynamical and structural analysis. Places are represented by circles and model resources or objects. Transitions are depicted as rectangles and represent events i.e. operations or activities that can change the system state. Arcs that are drawn either from places to transitions or from transitions to places and represent resource constraints and logical relationships between events.

In order to describe the dynamical behavior of a system in matter of its state and state changes, token (back dots) are used. Every place may hold either none or multiple tokens. The presence of a token in a place represents the current state of the modeled system.

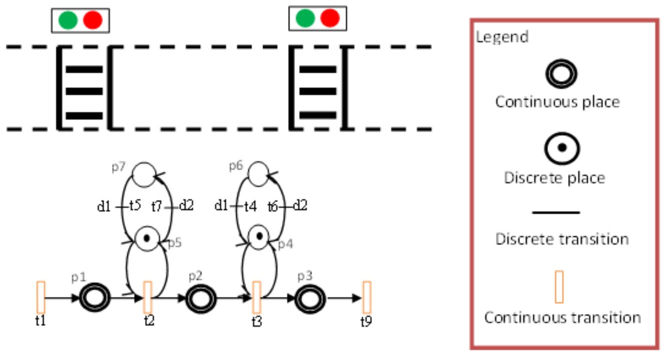

Fig. 1. Signalized road and the HPN model.

Formally, a Petri net is quadruplet $N = (P, T, Pre, Post)$ where P and T are, respectively, disjoint sets of places and transitions; Pre and $Post$: $P \times T \rightarrow N$ is the backward incidence matrix, $Post$: $T \times P \rightarrow N$ is the forward incidence matrix.

B. Generalized Batch Petri Nets

Hybrid Petri nets contain two parts: a discrete part and a continuous part since they include discrete and continuous places and transitions. For instance, in traffic system, traffic lights are modeled as discrete places and transitions and road sections can be modeled by continuous places as can be illustrated in (Fig. 1).

In (Fig. 1) we show an example of signalized road with its HPN model where $P^C = (p1, p2, p3)$ is the set of continuous places representing the three sections of the road, $P^D = (p4, p5, p6, p7)$ is the set of discrete places that models the signal system switching between red and green. $T^C = (t1, t2, t3, t9)$ is the set of continuous transitions model the limit between two sections and $T^D = (t4, t5, t7, t8)$ is the set of the discrete transitions which represent the delay associated with every section for switching between green and red lights.

Batches Petri nets are a modeling tool for systems with accumulation. The new with this extension of HPN is the introduction of batch concept; a batch (β_r) is a group of entities whose movement through a transfer zone (i.e. a portion of road), and is characterized by continuous variables, namely the length, the density, and the head position. Thus, batch places represent the delays on continuous flow and batch transitions behave like continuous transitions are associated with a maximum flow.

C. The triangular batches Petri net

In the following, some of the main definitions of triangular batches Petri nets are presented.

Definition 1. The triangular batches Petri net can be defined by the following elements:

A triangular batch Petri net is $6 - tuplet$ $N = (P, T, Pre, Post, \gamma, Time)$ where

P is a non-empty finite set of places partitioned into discrete (P^D), continuous (P^C) and batch places (P^B): $P = P^C \cup P^D \cup P^B$ Continuous places are used to model variables real values; While discrete places are used in modelling variables with integer values; Batch Places are used to model transfer zones whose elements move at the same speed

T is a non-empty finite set of transitions partitioned into discrete (T^D), continuous (T^C) and batch transitions (T^B).

Pre, respectively $Post$: $(P^D \times T \rightarrow N) \cup ((P^C \cup P^B) \times T \rightarrow R \geq 0)$ are defining the weight of arcs directed from places to transitions and the weight of arcs directed from transitions to places.

$\gamma : P^B \rightarrow \Re^4 \geq 0$ is the batch place function. It associates with each batch place $p_i \in P^B$ the $4 - uplet$ of continuous characteristics $(p_i) = (V_i, d_i^{max}, S_i, \Phi^{max})$ that represents, respectively, a speed, a maximum density, a length and a maximum flow.

$Time$: it represents the firing delay in case of discrete transitions and the maximal firing flow in case of continuous or batches transitions.

For analysis of Petri net, two notions are needed: Preset and Postset since they are crucial in modelling, respectively, pre-conditions and post-conditions of a system changes.

Definition 2 (Preset and Postset). Let $m = \mid P \mid$ and $n = \mid T \mid$ be the number of places and transitions in a TrBPN, respectively. The preset of a place p (resp. of a transition t) is defined as $\cdot p = \{t \in T \mid Post(p, t) > 0\}$ (resp. $\cdot t = \{p \in P \mid Pre(p, t) > 0\}$) and the postset is defined as $\cdot p = \{t \in T \mid Pre(p, t) > 0\}$ (resp. $\cdot t = \{p \in P \mid Post(p, t) > 0\}$).

Batches are moving at different speeds. For controlling this, the concept of controllable batch is introduced.

Definition 3 (Controllable batch). Additionally to the three basic characteristics of a batch, a controllable batch $(C\beta_r)$ is characterized also by the speed. $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau)) \in \Re^+ \times \Re^+ \times \Re^+ \times \Re^+$. An instantaneous batch flow of $(C\beta_r)(\tau)$ is given by: $\phi_r(\tau) = v_r(\tau) \cdot d_r(\tau)$.

The marking of the batch place is a series of batches ordered by their head positions.

The moving of batches and their evolution depend on the state of batch places and are governed by various functions, namely: Creation, Destruction, Merging and splitting.

Definition 4 (Dynamics of controllable batches). For every batch place, four continuous dynamic functions can change the values of its batches and then change its marking.

Creation: A controllable batch $C\beta_r(\tau) = (0, d_r(\tau), 0, v_r(\tau))$, is created inside a batch place p_i and added to its marking if its input flow is not null i.e. With

$$d_r(\tau) = \frac{\phi_i^{in}(\tau)}{v_i(\tau)}$$

Destruction: A controllable batch is destroyed, if its length is null i.e. $l_r(\tau) = 0$ and it is not a created batch i.e. $x_r(\tau) \neq 0$, such that $C\beta_r(\tau) = 0$ and removed from the marking

Merging: A controllable batch is obtained from the fusion of two batched, if they are in contact with the same density and the same speed. Consider the two controllable batches:

$(C\beta_r)(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ and $C\beta_h(\tau) = (l_h(\tau), d_h(\tau), x_h(\tau), v_h(\tau))$ with $x_r(\tau) = x_h(\tau) + l_r(\tau)$, $v_r(\tau) = v_h(\tau)$ and $d_r(\tau) = d_h(\tau)$, $(C\beta_r)(\tau)$ becomes $(C\beta_r)(\tau) = (l_r(\tau)+l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ and $C\beta_h(\tau)$is destroyed $C\beta_h = 0$

Splitting: A controllable can be split into two batches in contact with same density and the same speed

The batch Petri net that will be presented in section IV is based mainly on the same principles as triangular batch Petri net such as triangular fundamental diagram, generalizes the traffic flow modelling and tackles other cases of road networks. We extend the existing formalization to support, on one hand, dependencies between traffic flow dynamics and external conditions, and on the other hand, events that change dynamics at any place within roads.

III. Related works

Road traffic flow modeling has been investigated intensively [9], [20], [21] while approaching traffic networks as discrete event systems is taking much attention increasingly.

Among these graphical and mathematical tools, Petri Nets have been used for modelling and analyzing the behavior of traffic flow in road networks. Authors in [22] proposed a macroscopic traffic model based on continuous Petri nets where the number of cars in a given section is represented by the marking of a continuous place, the flow leaving the section is the firing speed of the related output transition.

The work in [23] has proposed Hybrid Petri net, which is a combination of continuous and discrete places and transitions, where the continuous part of the model represents the vehicle flow and the discrete part models the traffic light system at an intersection.

In [24], a timed hybrid petri net was introduced to model a signalized traffic intersection taking into account vehicle flow and dynamics, and the required time to cross the intersection.

Demongodin in [25] introduced an extended hybrid Petri net to model a transportation system with a variable speed limit that is called Generalized batches Petri net (GBPN). The proposed GBPN was applied in a giving portion of a highway through defining the batch node, the concept of batch that is a group of vehicles moving on different sections with continuous characteristics such as density, speed, position and length and the controllable batch which is a batch with a varying characteristic.

In order to model more accurately the vehicle flow and to reduce congestion, Gaddouri in [26] proposed the triangular batches Petri net (TrBPN), an extension of GBPN by redefining the batch place and adding a new parameter related to the maximum flow. This new place, called Bi-parts Batches place (BB-place), lead to define new flow-density relation that represents better the triangular fundamental diagram of road traffic flow [27] and governs the dynamics of controllable batches inside BB-places.

However, these models aren't able to express accurately some rules and situations in traffic applications such as priority roads, traffic that has to stop when approaching an intersection.

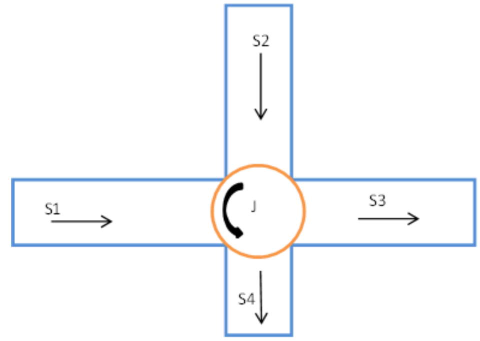

Fig. 2. Example of intersection with give way rule.

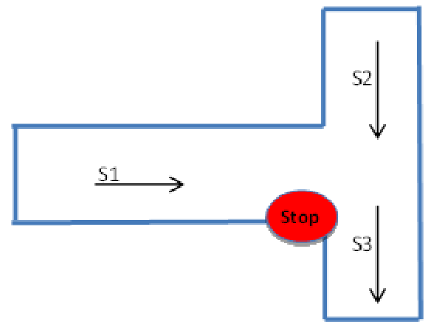

Fig. 3. Example of a controlled intersection with sign stop.

For example, the traffic behavior can change significantly due to the type of intersections whether they are right-of-way or roundabout, the presence of road signs or not. Therefore, taking into consideration non deterministic time based traffic dynamics will lead to improve modelling accuracy and generalization.

IV. Generalized nondeterministic batch Perti net (GNBPN)

The proposed model in this section builds upon the existing triangular batch Petri net [26] model and we show how it can deal with some complex cases of traffic networks that are illustrated with examples.

A. Motivation

Consider the following examples:

Example 1: In the intersection, when a batch approaches and aims to turn left, it must give way to the batch coming from the right and also wait for the batch, approaching from the opposite direction and reaching the intersection simultaneously, to move. A similar example is when entering vehicles must give way to traffic within the roundabout (Fig. 2).

Example 2: When there is a stop sign, a batch has always to stop even if the way is clear (Fig. 3).

By considering the above examples and other such situations the traffic behavior or the batch is not only depending on simple delay times or flow rules but can also depend on more complex uncontrollable events that may even change completely the batch characteristics. In the triangular batches Petri net modeling, these cases and others are not modeled precisely; For instance, in example 1, the batch doesn't have necessary the same behavior since the batch can be divided according to the other traffic behaviors, or are not taking into consideration as in example 2 where the condition that has to be filled is neither delay time nor flow rule, the traffic has to stop and wait for the way to be clear.

The purpose of this paper is to improve Petri Net efficiency and applicability in the urban and road traffic context. We propose then an extension of Triangular batch petri net, we named the extended formalism Generalized nondeterministic batch Perti net (GNBPN). It seeks to complete Petri net formalism by adding new non-deterministic time based transitions which we call untimed transition in order to formally represent changes that depend on external conditions(i.e. vehicles move at intersections only when the road ahead is empty). This allows us to include uncontrollable events which cannot be fixed in time. Moreover, we enrich the model to take into consideration relationships to previous observed changes at a given station and/or to neighboring roads at the same time; for instance, at fork, splitting a traffic flow into more accurate proportions can depend on temporal and spatial information (presence of an event, roadworks). The benefit of the added statistical feature is to enhance the firing rule of batch transitions.

In traffic road reality, the most common form of congestion is not localized like a wide moving jam, but spatially extended, and it often persists over several hours. In contrast to stop-and-go waves, the flow and velocity stay finite [28]. Adding to this, events don't lead always to change batch states, but more general; they lead that a batch can be burst into two different alternating batches with different characteristics. For example, an accident cannot always modify the maximum flows associated to the continuous and batch transition or vary the batch speed, but it can furthermore divide a batch into two batches with different states (free and congested). Therefore, knowing where the dynamics of traffic change i.e. where congestion occurs is crucial for alleviating traffic flow. The current Petri net formalism doesn't allow such information.

The model presented in this paper extends additionally the Petri Net class by introducing a new continuous characteristic: Ns the total number of segments for each batch place; Roads are divided into sections where the spatial variations in traffic flow can be neglected. The main advantages of adding this characteristic are:

1) Model traffic flow over a series of road segments, instead of model it at specific point (mostly, at the end of a road). Therefore, the dynamic evolution of traffic will be modeled more precisely;
2) Simplify modelling traffic road and make it more easier to analyze and locate more precisely events that change the traffic state.

B. GNBPN Formalism

Generalized nondeterministic batch Perti net (GNBPN) is defined formally as follows: $(P, T, Pre, Post, C, f, Prio, E)$

Where:

1) P, Pre and $Post$ are respectively the set of places, the pre-incidence and the post-incidence matrices as defined above in Def. 1;
2) T is a finite set of transitions that are partitioned into the three set of timed (T^t), untimed (T^u) and batch (T^B) transitions $T = T^t \cup T^u \cup T^B$;
3) C is the "characteristic function". It associates with every batch place three continuous characteristics $(V_i, d_i^{max}, S_i, N_s)$ that represent respectively speed, maximum density, and length and the total number of segments of the batch place;
4) f: is an application that associates a non-negative number to every transition:
   • if $t_j \in T^t$, then $f(t_j) = d_j$ denotes the firing delay associated with the timed transition expressed in time unit;
   • if $t_j \in T^B$, then $f(t_j) = \Phi_j$ denotes the maximal firing flow associated with the batch transition expressed in entities/time unit and estimated by statistical models in case of multiple outputs;
   We use the symbole f to refer to firing rules of transitions which can depend on both delay times and flow rules.
5) $Prio$ is the priority of a transition according to the output flow of a place, $\forall t_j \in T^u$. This priority's feature is added to support non-deterministic time based transitions.

The main contribution of this paper is summarized as follows:

- A definition of a new kind of non-deterministic time based transitions, untimed transitions. These transitions are essential to model operations that depend on external conditions (i.e. other places);
- Adding the total number of segments for every batch place as a new characteristic in order to increase the description exactness of the traffic dynamics and precision of where the change locates.

C. Illustrative examples

Example 1 (see Figure 2): We illustrate the primitives of the proposal petri net by the example below. It consists of a junction of two roads where a stop and give way is sited. The Sections S1 and S2 are represented respectively, by batch places p1, p2. The junction J is modelled by two batch places p31 and p32 that correspond respectively to the portions of the intersection on the opposite side of section 1 and section 2. While the maximum inputs flow of S1 and S2 are modelled by the batch transitions t1 and t2.

The priority rule of junction J is modelled by the untimed transition t3 and t4 representing that vehicles at sections S1 and S2 have to wait until the traffic ahead clears. The associated model is shown in Figure 4.

Example 2 (see Figure 3): This example models the scenario where vehicles in the road S1 have to stop at roads intersection due to stop sign and yield to vehicles coming from the section

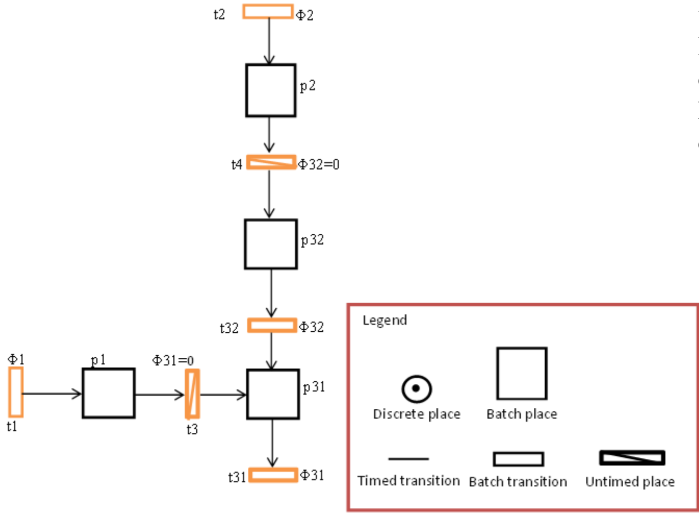

Fig. 4. Generalized nondeterministic batch Perti net for example 1.

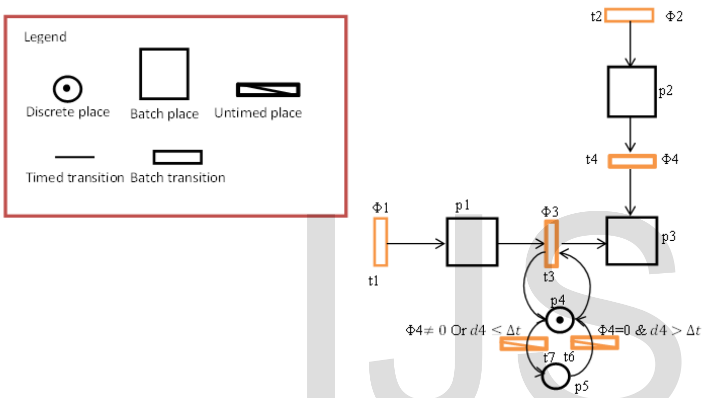

Fig. 5. Generalized nondeterministic batch Perti net for example 2.

S2. The Sections S1, S2 and S3 are represented respectively, by batch places p1, p2 and p3. The maximum input flow of S1 and S2 are modelled by the batch transitions t1and t2. Maximum flows $\Phi3$ and $\Phi4$ of batch transitions t3 and t4 represent, respectively, the maximum output flow of sections S1 and S2. Discrete places p4 and p5 represent the stopping state of the traffic and crossing the intersection. Untimed transitions t6 and t7 represent the stop at the end of section S1, i.e. stopping at least for a duration $d4 > \triangle t$ and wait for crossing vehicles to clear the intersection or cross otherwise. This case and others cannot be modelled by triangular batches Petri. The model in Figure 5 is designed to represent this example.

V. Conclusion and Future Works

This paper presents an extension of batch petri net to model the behavior of traffic systems particularly for complex cases such as intersections and priority roads. The extended model tackles the uncontrolled events problem by adding nondeterministic time based transitions that represent the dependencies between system dynamics and external conditions such as give way rules. Furthermore, we noticed that triangular batches Petri net modelling doesnt handle the case where events, that change the traffic state, can occur within roads. Thereby, we have proposed to divide roads into sections in order to locate events such as accidents and represent their effects on the systems behavior.

As a part of future research, we plan to:

- Focus on dynamic evolution of traffic flow and test the compliance of the proposed model with different practices of transportation system;
- Enhance firing rules by introducing statistical features to deal with spatial-temporal correlations of roads such as splitting a traffic flow at forks.

References

[1] OECD Transport Research Centre, *MANAGING TRAFFIC CONGESTION MANAGING TRAFFIC CONGESTION, ECMT European Conference of Ministers of Transport*, 2007.

[2] Cebr and INRIX, “The future economic and environmental costs of gridlock in 2030. An assessment of the direct and indirect economic and environmental costs of idling in road traffic congestion to households in the UK, France, Germany and the USA,” no. July, p. 67, 2014. [Online]. Available: http://inrix.com/economic-environment-cost-congestion/

[3] M. Picone, S. Busanelli, M. Amoretti, F. Zanichelli, and G. Ferrari, *Advanced Technologies for Intelligent Transportation Systems*, 2015.

[4] D. Schrank., B. Eisele., T. Lomax., and J. Bak., “2015 Urban Mobility Scorecard,” *Texas A&M Transportation Institue*, vol. 39, no. August, p. 5, 2015.

[5] “Environmental Handbook: Traffic Noise,” no. January, pp. 1–17, 2016.

[6] E. Bekiaris and Y J Nakanishi, *Economic Impacts of Intelligent Transportation Systems: Innovations and Case Studies*.

[7] D. Fuzhan Nasiri, A. H.F. Chow, and Y. Li, “Modeling and optimization of road transport facility operations,” *Journal of Facilities Management*, vol. 12, no. 3, pp. 268–285, 2014. [Online]. Available: http://dx.doi.org/10.1108/JFM-04-2013-0023

[8] D. R. Tobergte and S. Curtis, “Traffic Flow Modelling to Improve Traffic State Prediction,” *Journal of Chemical Information and Modeling*, vol. 53, no. 9, pp. 1689–1699, 2013.

[9] F. van Wageningen-Kessels, H. van Lint, K. Vuik, and S. Hoogendoorn, “Genealogy of traffic flow models,” *EURO Journal on Transportation and Logistics*, vol. 4, no. 4, pp. 445–473, 2014. [Online]. Available: http://link.springer.com/10.1007/s13676-014-0045-5

[10] L. Yang, “Stochastic Traffic Flow Modeling and Optimal Congestion Pricing,” 2012.

[11] a. Pascale, M. Nicoli, F. Deflorio, B. Dalla Chiara, and U. Spagnolini, “Wireless sensor networks for traffic management and road safety,” *IET Intelligent Transport Systems*, vol. 6, no. May 2011, p. 67, 2012.

[12] K. Nagel and M. Schreckenberg, “A cellular automaton model for freeway traffic,” pp. 2221–2229, 1992.

[13] A. Pascale, M. Nicoli, and U. Spagnolini, “Cooperative bayesian estimation of vehicular traffic in large-scale networks,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 15, no. 5, pp. 2074–2088, 2014.

[14] J. McCrea and S. Moutari, “A hybrid macroscopic-based model for traffic flow in road networks,” *European Journal of Operational Research*, vol. 207, no. 2, pp. 676–684, 2010. [Online]. Available: http://dx.doi.org/10.1016/j.ejor.2010.05.018

[15] M. J. Lighthill and G. B. Whitham, “On Kinematic Waves. II. A Theory of Traffic Flow on Long Crowded Roads,” *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 229, no. 1178, pp. 317–345, 1955. [Online]. Available: http://rspa.royalsocietypublishing.org/cgi/doi/10.1098/rspa.1955.0089

[16] P. I. Richards, “Shock Waves on the Highway,” *Technical Operations, Inc., Arlington, Massachusetts*, pp. 42 – 51, 1956.

[17] B. Haut, G. Bastin, and Y. Chitour, “A macroscopic traffic model for road networks with a representation of the capacity drop phenomenon at the junctions,” *IFAC Proceedings Volumes (IFAC-PapersOnline)*, vol. 16, no. 1995, pp. 114–119, 2005.

[18] Le Bail, H. Alla, and R. David, "Hybrid petri nets," in *European Control Conference*, Grenoble, France, 1991, pp. 1472–1477.
[19] R. David and H. Alia, *Discrete, continuous, and hybrid petri nets*, 2005.
[20] S. P. Hoogendoorn and P. H. L. Bovy, "State-of-the-art of vehicular traffic flow modelling," *Proceedings of the I MECH E Part I Journal of Systems & Control in Engineer*, vol. 215, no. 4, pp. 283–303, 2001.
[21] R. Mohan and G. Ramadurai, "State-of-the art of macroscopic traffic flow modelling," *International Journal of Advances in Engineering Sciences and Applied Mathematics*, vol. 5, no. 2-3, pp. 158–176, 2013. [Online]. Available: http://link.springer.com/10.1007/s12572-013-0087-1
[22] J. J. Julvez and R. K. Boel, "A continuous petri net approach for model predictive control of traffic systems," *IEEE Transactions on Systems, Man, and Cybernetics Part A:Systems and Humans*, vol. 40, no. 4, pp. 686–697, 2010.
[23] C. R. Vázquez, H. Y. Sutarto, R. Boel, and M. Silva, "Hybrid Petri net model of a traffic intersection in an urban network," *Proceedings of the IEEE International Conference on Control Applications*, pp. 658–664, 2010.
[24] J. Wang, J. Yan, and L. Li, "Microscopic Modeling of a Signalized Traffic Intersection Using Timed Petri Nets," *IEEE Transactions on Intelligent Transportation Systems*, vol. 17, no. 2, pp. 305–312, 2016.
[25] I. Demongodin, "Modelling and analysis of transportation networks using batches Petri nets with controllable batch speed," in *Applications and Theory of Petri Nets*, 2009, pp. 204–222.
[26] R. Gaddouri, L. Brenner, and I. Demongodin, "Extension of Batches Petri Nets by Bi-parts batch places."
[27] Transportation Research Board, "75 Years of the Fundamental Diagram for Traffic Flow Theory. Greenshields Symposium," *Transportation Research Circular*, vol. E-C149, no. June, pp. 45–62, 2011. [Online]. Available: http://onlinepubs.trb.org/onlinepubs/circulars/ec149.pdf#page=53
[28] M. Schönhof and D. Helbing, "Empirical Features of Congested Traffic States and Their Implications for Traffic Modeling," *Transportation Science*, vol. 41, no. 2, pp. 135–166, 2007.